Gröbner Bases in Cryptography through the example of the Rank Decoding Problem

Maxime Bros

Algebraic Rewriting Seminar

April 15th, 2021

Our Attacks

Magali Bardet, Pierre Briaud, Maxime Bros, Philippe Gaborit, Vincent Neiger, Olivier Ruatta, and Jean-Pierre Tillich.
An algebraic attack on rank metric code-based cryptosystems.
In *EUROCRYPT 2020*, pages 64–93. Springer, 2020.

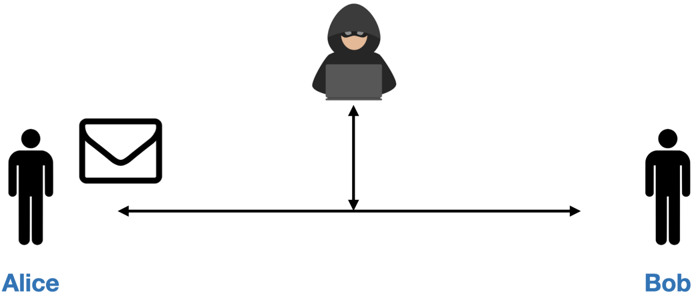
Magali Bardet, Maxime Bros, Daniel Cabarcas, Philippe Gaborit, Ray Perlner, Daniel Smith-Tone, Jean-Pierre Tillich, and Javier Verbel.
Improvements of algebraic attacks for solving the rank decoding and minrank problems.
In *ASIACRYPT 2020*, pages 507–536. Springer, 2020.

Notation

- $\mathbb{F}_q$: finite field with q elements (usually $q = 2$, sometimes $q = p^r$),
- $\mathbb{F}_{q^m}$ is its extension of degree m,
- $\mathbb{F}_{q^m}$ is also a finite field with q^m elements,
- $\mathbb{F}_{q^m}$ can be seen as an $\mathbb{F}_q$-vector space of dimension m,

- $a \xleftarrow{\$} [\![1, 2, \ldots, N]\!]$.

Cryptography



Cryptography

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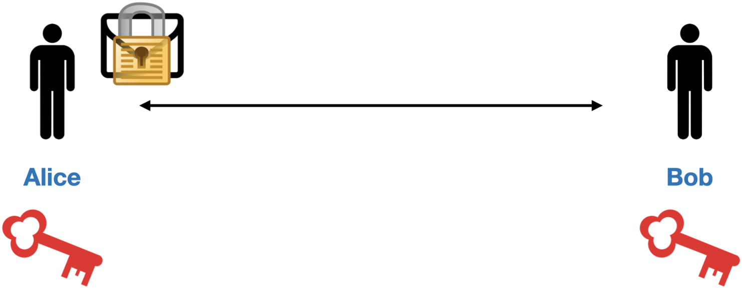

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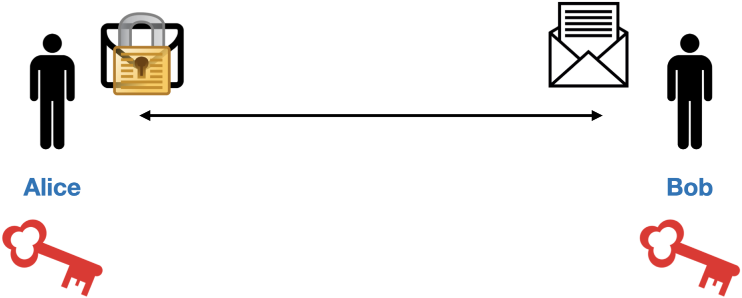

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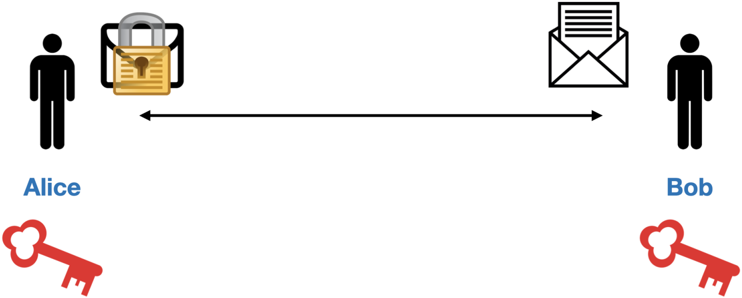
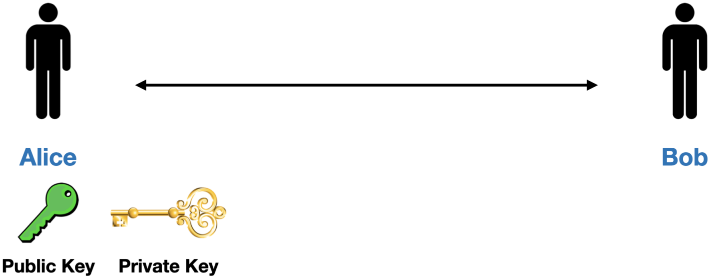


Many limitations due to keys' exchanges

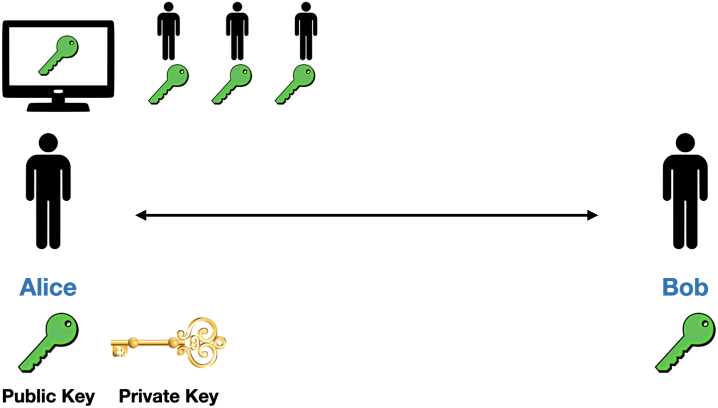
Assumption

There exists such a **symmetric** cryptographic scheme (e.g. AES).

Asymmetric Cryptography



Asymmetric Cryptography



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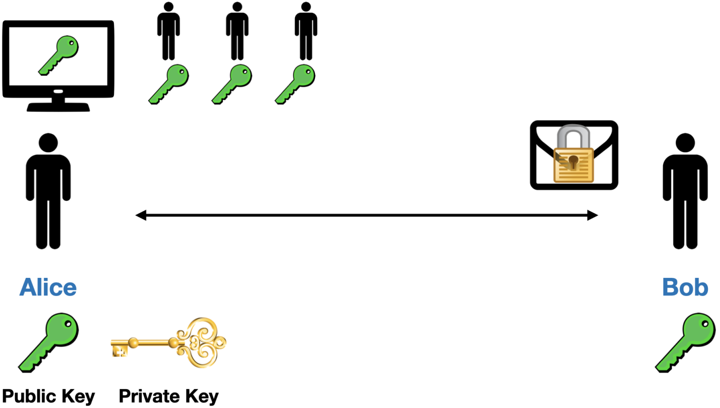

Asymmetric Cryptography

Asymmetric Cryptography

Revolution dates

- Merkle: 1975
- First protocol in 1976:

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. IT-22,

New Directions in Cryptography

Invited Paper

WHITFIELD DIFFIE AND MARTIN E. HELLMAN, MEMBER, IEEE

The First Protocol

Let p be a (big) prime, let us consider

$$G = \{1, g, g^2, \ldots, g^{p-2}\} \quad (= (\mathbb{Z}/p\mathbb{Z})^\times)$$

- Alice: $a \xleftarrow{\$} [\![0, 1, \ldots, p-2]\!]$ (long term)
- Bob: $r \xleftarrow{\$} [\![0, 1, \ldots, p-2]\!]$ (single use)

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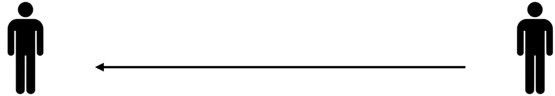
$$K = (g^a)^r$$



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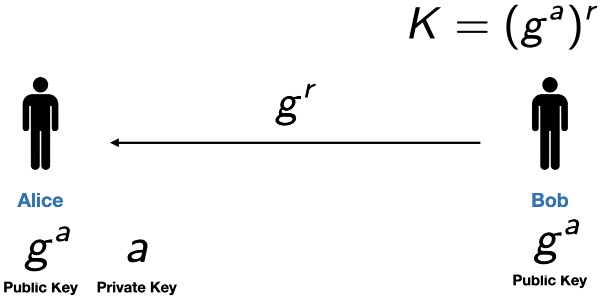

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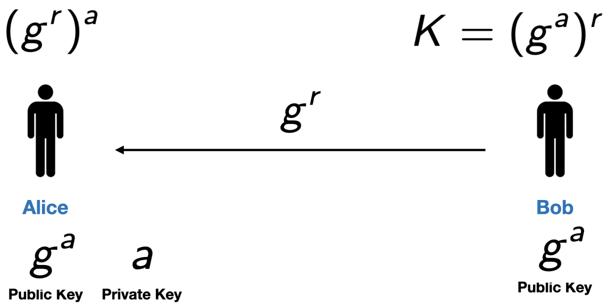

Quantum Threat and a Worldwide Competition

- Diffie-Hellman key exchange protocol.
- It relies on the hardness of the **Discrete Logarithm** problem.
- RSA (integer factorization) and DH (discrete logarithm) are widely used nowadays.
- Similar complexities and most importantly: solved by Shor's algorithm (1994).
- **NIST Standardization Process** was announced in 2015-2016, first deadline at the end of 2017.
- 5 new families of problem: lattice, coding theory, multivariate systems, isogeny, hash functions.

A simple problem in linear algebra

Let $k < n$ be integers, $H \in \mathbb{F}_q^{(n-k)\times n}$, $e \in \mathbb{F}_q^{n\times 1}$, and $s \in \mathbb{F}_q^{(n-k)\times 1}$.

$$\begin{pmatrix} & \mathrm{H} & \end{pmatrix} \begin{pmatrix} \\ e \\ \\ \end{pmatrix} = \begin{pmatrix} s \end{pmatrix}$$

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- One easily finds one or several solutions for e.

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- One easily finds one or several solutions for e.
- Therefore, one can not control the **weight** of e for a given metric!

Decoding Problem

Definition (Syndrome Decoding (SD) Problem - computational version)

Input: a parity-check matrix $H \in \mathbb{F}_{q^m}^{(n-k)\times n}$ of a code $\mathcal{C}$ (i.e. a subspace of $\mathbb{F}_{q^m}^n$), an integer $r \in \mathbb{N}$ and a vector $s \in \mathbb{F}_{q^m}^{n-k}$.
Output: a vector $e \in \mathbb{F}_{q^m}^n$ such that $He^T = s^T$ and $w(e) \leq r$.

Definition (Decoding Problem - computational version)

Input: a code $\mathcal{C}$ (i.e. a subspace of $\mathbb{F}_{q^m}^n$), an integer $r \in \mathbb{N}$ and a vector $y \in \mathbb{F}_{q^m}^n$.
Output: $c \in \mathcal{C}$ such that $w(y-c) = w(e) \leq r$.

Decoding Problem $\Longleftrightarrow$ Syndrome Decoding Problem

- Euclidian metric $\Longrightarrow$ lattice-based cryptography
- Hamming metric $\Longrightarrow$ code-based cryptography
- Rank metric $\Longrightarrow$ rank-based cryptography

Introduction to Coding Theory

Error correcting codes are used to transmit informations (satellites, DVD, ...) **and also for cryptographic purpose!**

Definition (Code)

A code $\mathcal{C}$ is vector space of $GF(q)^n$ of dimension k.

$$\begin{array}{rcl} \mathcal{E}: GF(q)^k & \longrightarrow & GF(q)^n \\ m & \longmapsto & mG \end{array}$$

$$G = k\left\{ \overbrace{\left(\begin{array}{c:c} I_k & A \end{array}\right)}^{n} \right.$$

$$\underbrace{\begin{pmatrix} m \end{pmatrix}}_{k}\left(\begin{array}{c:c} \underbrace{m}_{k} & \underbrace{red}_{n-k} \end{array}\right)$$

Reminder about Error Correcting Codes

Definition (Parity Check Matrix)

H is a parity check matrix for the code $\mathcal{C}$ if for every word $c \in GF(q)^n$:

$$c \in \mathcal{C} \iff Hc^T = 0_{n-k}.$$

Summary

- G is the generator matrix, H the parity-check matrix of a code.
- Compute mG to encode the message m, send it.
- The receiver gets $y = mG + e$, e "small"

$$\Longrightarrow \text{decoding instance.}$$

- Alternatively, he computes

$$Hy^T = H(mG + e) = Hc^T + He^T = s$$

$$\Longrightarrow \text{syndrome decoding instance.}$$

Rank Decoding Problem

Definition (Decoding Problem for $\mathbb{F}_{q^m}$-linear codes)

Input: a code $\mathcal{C}$ which is a subspace of $(\mathbb{F}_{q^m})^n$ of dimension k, and a vector $y = c + e$ where $c \in \mathcal{C}$ and $\text{Rank}(e) = r \in \mathbb{N}$.
Output: c.

Remark: the metric considered here is the **Rank metric** in $(\mathbb{F}_{q^m})^n$.

Rank metric on a toy example

Let $B = \{1, \alpha, \alpha^2, \alpha^3\}$ be a basis of $\mathbb{F}_{2^4}$ seen as an $\mathbb{F}_2$-vector space; $\alpha^4 = \alpha + 1$.

$$v := \begin{pmatrix} \alpha^4 & 1 & \alpha^4 & 0 & \alpha \end{pmatrix} \in (\mathbb{F}_{2^4})^5$$

$$\text{Mat}(v) := \begin{array}{c} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \end{array} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in (\mathbb{F}_2)^{4\times 5}$$

$$\text{Rank}(v) := \text{Rank}(\text{Mat}(v)) = 2.$$

Algebraic attack

- **Algebraic Attack:** one models a problem with a **system of algebraic equations** and solves it.
- In cryptanalysis, the (often unique) solution to this system of equations can be the **private key** or the **plaintext**.
- For the Rank Decoding problem, the solution is the small rank error e;
- Classic approaches:
  - **generic** Gröbner basis (GB) algorithms
  - **specific** Linearization techniques

Gröbner Basis algorithms

System of equations

$\{f_1, \ldots, f_m\} \in \mathbb{F}_q[x_1, \ldots, x_n]$

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = 0 \\ f_2(x_1, x_2, \ldots, x_n) = 0 \\ \vdots \\ f_m(x_1, x_2, \ldots, x_n) = 0 \end{cases}$$

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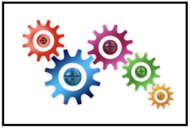
Gröbner basis algorithm



Solution

$$\begin{cases} x_1 = c_1 \in \mathbb{F}_q \\ x_2 = c_2 \in \mathbb{F}_q \\ \vdots \\ x_n = c_n \in \mathbb{F}_q \end{cases}$$

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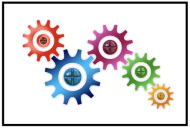

$$\mathcal{O}\left(\binom{n+d}{d}^{2.807}\right)$$

Solution

$$\begin{cases} x_1 = c_1 \in \mathbb{F}_q \\ x_2 = c_2 \in \mathbb{F}_q \\ \vdots \\ x_n = c_n \in \mathbb{F}_q \end{cases}$$

Gröbner Basis Complexity

Let us consider this system with quadratic polynomials in $\mathbb{F}_2[x, y, z]$

$$F := \begin{cases} f_1 = xy + xz \\ f_2 = y^2 + yz \\ f_3 = x^2 + yz + 1 \end{cases}$$

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One wants to compute S-polynomials.
$\Longrightarrow$ Macaulay matrix of a system at a given degree.

$$\mathcal{M}_{F,2} = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \end{array} \begin{array}{c} \begin{array}{ccccccc} x^2 & xy & xz & y^2 & yz & z^2 & 1 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

Gröbner Basis Complexity

$$\mathcal{M}_{F,3} =$$

$$\begin{array}{c c}
 & \begin{array}{ccccccccccccc} x^3 & x^2y & x^2z & xy^2 & xyz & xz^2 & y^3 & y^2z & yz^2 & z^3 & x & y & z \end{array} \\
\begin{array}{c} xf_1 \\ xf_2 \\ xf_3 \\ yf_1 \\ yf_2 \\ yf_3 \\ zf_1 \\ zf_2 \\ zf_3 \end{array} &
\begin{pmatrix}
 & 1 & 1 & & & & & & & & & & \\
 & & & 1 & 1 & & & & & & & & \\
1 & & & & 1 & & & & & & 1 & & \\
 & & & 1 & 1 & & & & & & & & \\
 & & & & & & 1 & 1 & & & & & \\
 & 1 & & & & & & 1 & & & & 1 & \\
 & & & & 1 & 1 & & & & & & & \\
 & & & & & & & 1 & 1 & & & & \\
 & & 1 & & & & & & 1 & & & & 1
\end{pmatrix}
\end{array}$$

Gröbner Basis Complexity

$$\widetilde{\mathcal{M}_{F,3}} =$$

$$\begin{array}{c}
\begin{array}{ccccccccccccc}
x^3 & x^2y & x^2z & xy^2 & xyz & xz^2 & y^3 & y^2z & yz^2 & z^3 & x & y & z
\end{array} \\
\begin{pmatrix}
 & 1 & & & & & & & 1 & & & & 1 \\
 & & & 1 & & 1 & & & & & & & \\
1 & & & & & 1 & & & & & 1 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 1 & & 1 & & & & \\
 & & 1 & & & & & & 1 & & & & 1 \\
 & & & & 1 & 1 & & & & & & & \\
 & & & & & & & 1 & 1 & & & & \\
 & & & & & & & & & & & 1 & 1
\end{pmatrix}
\end{array}$$

Our Previous Attack

System of equations

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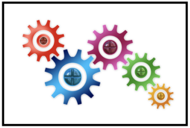
Additional equations

Gröbner basis algorithm



$$\mathcal{O}\left(\binom{n+d}{d}^{2.807}\right)$$

Solution

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d

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Linearization

- Sometimes the number of equations is greater than the number of **distinct monomials** that appear in the system.
- This allows one to solve the system directly by **linearization**.
- Thus, one only has to solve a huge **linear system** and **no longer requires generic GB algorithms**.
- Moreover, one can take advantage of the sparsity of the system to use Wiedemann's algorithm instead of Strassen's.

Linearization toy example

$$\begin{cases} f_1 = xz + yz + z \\ f_2 = yz + z + 1 \\ f_3 = xyz + xz + 1 \\ f_4 = xyz + z + 1 \end{cases}, \quad \in \mathbb{F}_2[x, y, z].$$

- We want to find the only point $(x_0, y_0, z_0) \in (\mathbb{F}_2)^3$ where all these polynomials vanish.
- 20 dictinct monomials of degree less than or equal to 3 in $\mathbb{F}_2[x, y, z]$.
- **Nevertheless, only 5 of them appear in this system of 4 equations.**
  $\Longrightarrow$ One looks for a vector in the right kernel of the form $(c_1, c_2, c_3, c_4, 1)^\top$

$$\begin{array}{c} \begin{array}{ccccc} xyz & xz & yz & z & 1 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array} \Longrightarrow \begin{cases} xyz = c_1 \\ xz = c_2 \\ yz = c_3 \\ z = c_4 \end{cases}$$

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Our Modeling for the Rank Decoding problem

- Recall that we receive the word $y = c + e$ where $c \in \mathcal{C}$ and $\text{Rank}(e) = r$.
- New code: $\widetilde{\mathcal{C}} = \mathcal{C} + \langle y \rangle$ contains all non-zero multiples λe, $\forall \lambda \in \mathbb{F}_{q^m}^{\times}$.
- Let H be a parity-check matrix of $\widetilde{\mathcal{C}}$.
- Since **all words of rank** r in $\widetilde{\mathcal{C}}$ are multiples of e, we want to solve the following equation:

$$(S_1 \; S_2 \; \ldots \; S_r)\, CH^\top = (0).$$

Remark: the entries of C are in $\mathbb{F}_q$

$$\underbrace{(S_1 \; S_2 \; \ldots \; S_r)}_{e'} \left(CH^\top\right) = (0).$$

- $e' \neq 0 \in \text{Ker}\left(CH^\top\right) \implies \text{Rank}\left(CH^\top\right) \leq r - 1$.
  Thus, all maximal minors of $CH^\top$ vanish.
- This modeling (based on Ourivksi and Johansson's one) is considered in Bardet and al., EUROCRYPT 2020.

Our Modeling for the Rank Decoding problem

Proposition (Maximal Minors of $CH^\top$)

The Maximal Minors of $CH^\top$ are polynomials over $\mathbb{F}_{q^m}$ of the form

$$\sum_{T\subset\{1..n\},\#T=r} (-1)^{f(T)} \det(H)_T \underbrace{\det(C)_T}_{:=c_T}$$

Proof: Cauchy-Binet's formula that generalizes the formula for determinant of square matrices: $\det(AB) = \det(A)\det(B)$ ■

- Fact 1: consider $\det(C)_T$ as new variables c_T's
  $\Longrightarrow$ **It yields to a linear system in the c_T's.**
- Fact 2: specialization I_r in C.

$$C = \left[\begin{array}{c|cccc} & C_{1,1} & C_{1,2} & \dots & C_{1,(n-r)} \\ I_r & \vdots & \vdots & \vdots & \vdots \\ & C_{r,1} & C_{r,2} & \dots & C_{r,(n-r)} \end{array}\right]$$

$\Longrightarrow$ **Those new variables include the coefficients of C!**

- This is called the **MaxMinors** modeling.
- Note that we consider **determinants** (instead of monomials only) as new (linearization) variables!

Complexity of our attack against Rank Decoding

Recall that we have a **linear system** in the variables c_T's arising from the vanishing of maximal minors of $CH^\top$.

$$\begin{cases} \binom{n}{r} - 1 & \text{variables } c_T\text{'s (in } \mathbb{F}_q), \\ m\binom{n-k-1}{r} & \text{equations over } \mathbb{F}_q. \end{cases}$$

Complexity of our algorithm against RD: **overdetermined case**

When $m\binom{n-k-1}{r} \geq \binom{n}{r} - 1$,

$$\mathcal{O}\left(m\binom{n-k-1}{r}\binom{n}{r}^{\omega-1} \right).$$

Remark: **this linear system is not dense at all, it is sparse, but not sparse enough to benefit from using the Wiedemann approach!**

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Super-overdetermined case

One chooses **the biggest integer p** so that

$$m\binom{n-k-1-p}{r} \geq \binom{n-p}{r} - 1$$

$$\Longrightarrow \quad \mathcal{O}\left(m\binom{n-k-1-p}{r}\binom{n-p}{r}^{\omega-1}\right)$$

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Hybrid case

One chooses **the smallest integer a** so that

$$m\binom{n-k-1}{r} \geq \binom{n-a}{r} - 1$$

$$\Longrightarrow \quad \mathcal{O}\left(q^{ar} m \binom{n-k-1}{r} \binom{n-a}{r}^{\omega-1}\right)$$

Comparison with previous Attacks

- Attack in k bits $\Longrightarrow$ Require 2^k bit-operations,
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	(m, n, k, r)	$\frac{m\binom{n-k-1}{r}}{\binom{n}{r}-1}$	a	p	**Def.**	**Prev.**	**Last**
ROLLO-I-128	$(79, 94, 47, 5)$	1.97	0	9	**128**	**117**	**71**
ROLLO-I-192	$(89, 106, 53, 6)$	1.06	0	0	**192**	**144**	**87**
ROLLO-I-256	$(113, 134, 67, 7)$	0.67	3	0	**256**	**197**	**151***
ROLLO-II-128	$(83, 298, 149, 5)$	2.42	0	40	**128**	**134**	**93**
ROLLO-II-192	$(107, 302, 151, 6)$	1.53	0	18	**192**	**164**	**111**
ROLLO-II-256	$(127, 314, 157, 7)$	0.89	0	6	**256**	**217**	**159***
ROLLO-III-128	$(101, 94, 47, 5)$	2.52	0	12	**128**	**119**	**70**
ROLLO-III-192	$(107, 118, 59, 6)$	1.31	0	4	**192**	**148**	**88**
ROLLO-III-256	$(131, 134, 67, 7)$	0.78	0	0	**256**	**200**	**131***
RQC-I	$(97, 134, 67, 5)$	2.60	0	18	**128**	**123**	**77**
RQC-II	$(107, 202, 101, 6)$	1.46	0	10	**192**	**156**	**101**
RQC-III	$(137, 262, 131, 7)$	0.93	3	0	**256**	**214**	**144**

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"Despite the development of algebraic attacks, NIST believes rank-based cryptography should continue to be researched. The rank metric cryptosystems offer a nice alternative to traditional hamming metric codes with comparable bandwidth."